Asymptotic bounds for energy of binary codes

Peter Boyvalenkov, Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences, 8 G.Bonchev str., 1113 Sofia, Bulgaria

Joint wok with P. Dragnev (Indiana-Purdue University, Fort Wayne), D. Hardin (Vanderbilt University, Nashville), E. Saff (Vanderbilt University, Nashville), M. Stoyanova (Sofia University)

(Based on "Energy bounds for codes and designs in Hamming spaces", to appear in Designs, Codes and Cryptography.)

We refer to any nonempty set $C \subset H(n,2)$ as a *binary code*. For a given potential function $h : [-1,1) \to (0,+\infty)$, we define the *h-energy* of C by $E(n,C;h) := \frac{1}{|C|}\sum_{x,y\in C, x\neq y} h(\langle x,y\rangle)$, where $\langle x,y\rangle = 1-2d(x,y)/n$, $d(x,y)$ is the Hamming distance between x and y. Many problems of interest can be formulated as minimizing the quantity $E(n,C;h)$ for a suitable h over codes C of fixed cardinality; that is, to determine $\mathcal{E}(n,M;h) := \min\{E(n,C;h) : |C| = M\}$, the minimal h-energy of a code $C \subset H(n,2)$ of cardinality M.

Recently, we derived an universal lower bound on $\mathcal{E}(n,M;h)$. In this talk we will consider the behaviour of our bound in the asymptotic process where the strength τ is fixed, and the dimension n and the cardinality M_n tend simultaneously to infinity in certain relation.

Theorem. We have

$$\lim_{n\to\infty} M_n \left(\sum_{i=0}^{k-1} \rho_i h(\alpha_i) - \sum_{j=0}^{k-1} \frac{h^{(2j)}(0)}{(2j)!} \cdot b_{2j} \right) = \delta_k^{2k-1} \left(h\left(-\frac{1}{\delta_k}\right) - R\left(-\frac{1}{\delta_k}\right) \right) - R(1),$$

where $R(t) := \sum_{j=0}^{2k-1} \frac{h^{(j)}(0)}{j!} t^j$, k and δ_k are constants.

The participating parameters and their behaviour will be explained in the talk.